CONVERGENT DETONATION WAVES UNDER CHAPMAN-JOUGUET CONDITIONS IN MEDIA WITH VARIABLE AND CONSTANT INITIAL DENSITIES

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We consider the flow of gas acted on by a detonation wave converging at a constant rate under Chapman-Jouguet conditions in a medium with a variable initial density. The dependence of the character of flow before and after focusing of the wave on the density distribution and on the heat capacity ratio is investigated.

In Section 2 we consider convergent and divergent self-similar detonation waves in a medium with a constant initial density. The self-similarity index, which turns out to be nonunique, is determined.

1. Let a strong detonation wave move towards the center of symmetry in a gas of variable density $\rho_0 = Ar^{-\omega}$ (A is a constant, r is the distance to the center of symmetry). The velocity D of the detonation wave depends on the energy influx Q into a unit mass of gas at the wave front. We consider both of these quantities constant.

Following [1], we can represent the velocity v, the density ρ, and the pressure p as

$$v = \frac{r}{t} V(\lambda), \qquad \rho = \frac{A}{r^{\omega}} R(\lambda), \qquad p = \frac{A}{r^{\omega-2} t^2} P(\lambda), \qquad \lambda = \frac{r}{D\,|t|} \tag{1.1}$$

The motion of the gas in the perturbed zone is described by Eqs. [1]

$$\frac{dZ}{dV} = \frac{Z\,\{2(V-1)^3 + (\nu-1)(\gamma-1)V(V-1)^2 - [2(V-1) + \omega(\gamma-1)/\gamma]\,Z\}}{(V-1)\,[V(V-1)^2 + (\omega/\gamma - \nu V)\,Z]} \tag{1.2}$$

$$\frac{d\ln\lambda}{dV} = \frac{Z-(V-1)^2}{V(V-1)^2 + (\omega/\gamma - \nu V)\,Z} \qquad \left(Z = \frac{\gamma P}{R}\right) \tag{1.3}$$

$$ZR^{1-\gamma} = C_1\,[R(V-1)]^{\frac{\omega(\gamma-1)}{\nu-\omega}}\,\lambda^{-2} \tag{1.4}$$

Here γ is the ratio of heat capacities, $\nu = 1, 2, 3$ for plane, cylindrical and spherical symmetry, respectively and the constant C_1 can be determined from the initial conditions.

In the event of fulfillment of the Chapman-Jouguet conditions at the front of a strong detonation wave we have the initial conditions (the values of the parameters at the detonation wave are denoted by the subscript 2)

$$V_2 = \frac{1}{\gamma+1}, \quad Z_2 = \frac{\gamma^2}{(\gamma+1)^2}, \quad R_2 = \frac{\gamma+1}{\gamma} \quad \text{for} \quad \lambda_2 = 1 \tag{1.5}$$

Focusing of the detonation wave coincides with the instant $t = 0$.

In order to solve the problem we must construct the integral curve on the ZV plane which emerges from the initial point (1.5) for $\lambda = 1$ and arrives at the point $O(0, 0)$ at $\lambda = \infty$, where λ must vary monotonously along the curve. Analysis of Eqs. (1.2) and (1.3) implies that a convergent detonation wave under Chapman-Jouguet conditions is possible only for $\omega \geq (\nu - 1)\gamma/(\gamma + 1)$.

Let us consider the spherical case ($\nu = 3$). For $\omega > 2\gamma/(\gamma + 1)$ the derivative $dZ/dV > 0$ at the initial point (1.5). For $\omega = 2\gamma/(\gamma + 1)$ the initial point is singular and is associated with one positive and one negative value of dZ/dV. In order to make $\omega = 2\gamma/(\gamma + 1)$ the limiting case, we take just the positive of the two values of dZ/dV for $\omega > 2\gamma/(\gamma + 1)$; only one integral curve emerges from the singular point in this direction.

Numerical solution of Eqs. (1.2) and (1.3) on a computer indicated that in the case $\omega = 2\gamma/(\gamma + 1)$, depending on the value of γ, the gas flow before and after the instant of detonation wave focusing can occur in one of five states. Each state is characterized by a distinctive position of the integral curve emerging from initial point (1.5) relative to the singular points of Eq. (1.2). The character of the singular points of Eq. (1.2) is described in [1].

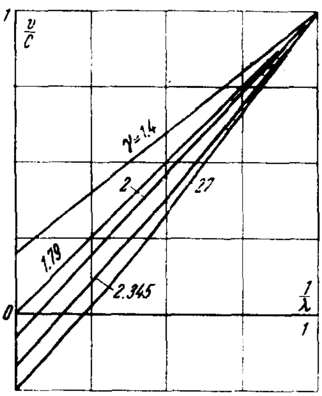

Fig. 1

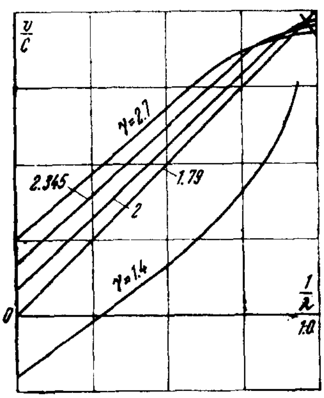

Fig. 2

State A for $1 < \gamma < 1.790$. At the instant $t = 0$ the velocity of the gas is the same throughout the space and is directed towards the center. For $t > 0$ a shock wave moves through the gas away from the center of symmetry. The intensity of the shock wave diminishes as γ approaches 1.790.

State B for $\gamma = 1.790$. At the instant $t = 0$ the velocity of the gas becomes zero everywhere in the space, and the shock wave which arises at $t = 0$ for smaller values of γ becomes a weak discontinuity.

State C for $1.790 < \gamma < 2.345$. For $t = 0$ the velocity of the gas is the same everywhere in the space and is directed away from the center. The direction of the velocity changes under the influence of the pressure gradient. The weak discontinuity is manifested as discontinuities of the derivatives of higher order than the first.

State D for $\gamma = 2.345$. For $t = 0$ the velocity of the gas is the same everywhere in the space and is directed away from the center. Whereas in the above cases the phase velocity of the particles filling the space can be either supersonic or subsonic for $t > 0$, in this case the phase velocities are supersonic only.

State E for $\gamma > 2.345$. For $t > 0$ a void arises at the center of symmetry and propagates with a certain velocity.

A similar breakdown of flow states occurs for $\omega > 2\gamma/(\gamma + 1)$.

We note that the acceleration of the gas particles at the detonation wave front is infinitely large for $\omega > 2\gamma/(\gamma + 1)$; for $\omega = 2\gamma/(\gamma + 1)$ the acceleration is finite.

The curves of velocity distribution in the stream for $\omega = 2\gamma/(\gamma + 1)$ and for various values of γ in the cases $t < 0$ and $t > 0$ appear in Figs. 1 to 3. The vertical axis represents v/c, where $c = (\gamma + 1)^{-1} D \cdot \operatorname{sgn} t$; the horizontal axis represents either λ or $1/\lambda$. The velocities directed towards the center of symmetry are considered negative; the velocities directed away from the center of symmetry are considered positive.

Fig. 1 represents flow up to the instant of focusing of the detonation wave ($t < 0$); Figs. 2 and 3 represent flow after the instant of focusing ($t > 0$).

2. The problem of propagation of a strong convergent detonation wave in a gas with a constant initial velocity is considered in [2]. In this section we solve the problem by the method described in [1].

Let the front of the strong detonation wave converge in accordance with the law

$$r_2 = a|t|^\delta \qquad (2.1)$$

We can write the velocity v, the density ρ, and the pressure p as

$$v = \frac{r}{t} V(\xi), \qquad \rho = \rho_0 R(\xi), \qquad p = \rho_0 \frac{r^2}{t^2} P(\xi), \qquad \xi = \frac{r}{a|t|^\delta} \qquad (2.2)$$

The equations of motion [1] then become

$$\frac{dZ}{dV} = \frac{Z\{[2(V-1)+\nu(\gamma-1)V](V-\delta)^2 - (\gamma-1)V(V-1)(V-\delta) - [2(V-1)+2\gamma^{-1}(1-\delta)(\gamma-1)]Z\}}{(V-\delta)\{V(V-1)(V-\delta)+[2\gamma^{-1}(1-\delta)-\nu V]z\}} \qquad (2.3)$$

$$\frac{d\ln\xi}{dV} = \frac{Z-(V-\delta)^2}{V(V-1)(V-\delta)+[2\gamma^{-1}(1-\delta)-\nu V]Z} \qquad (2.4)$$

$$ZR^{1-\gamma} = C_1[R(V-\delta)]^{\frac{2(\delta-1)}{\nu\delta}} \xi^{-\frac{2}{\delta}} \qquad (2.5)$$

Here we assume that Q is proportional to D^2, that both of these quantities are variable, and that the Chapman-Jouguet conditions are fulfilled. The following Eqs. are then fulfilled at the front of the strong detonation wave:

$$V_2 = \frac{\delta}{\gamma+1}, \qquad Z_2 = \frac{\delta^2\gamma^2}{(\gamma+1)^2}, \qquad R_2 = \frac{\gamma+1}{\gamma} \qquad \text{for } \xi_2 = 1 \qquad (2.6)$$

Analysis of Eqs. (2.3) and (2.4) indicates that convergent and divergent detonation waves are possible under the following respective conditions:

$$0 < \delta \leqslant \delta_*, \qquad \delta_* \leqslant \delta \leqslant 1 \qquad \left(\delta_* = \frac{3(\gamma+1)}{\gamma(2+\nu)+3}\right) \qquad (2.7)$$

The parabola $Z = (V-\delta)^2$ in the plane ZV has two singular points. For $\delta_* \leq \delta \leq 1$, i.e. in the case of a divergent detonation wave, initial point (2.6) lies between these singular points on the parabola $Z = (V-\delta)^2$. The integral curve in this case passes through the upper singular point (node) of the parabola $Z = (V-\delta)^2$; it must then be extended to the infinitely distant singular point (the saddle point $Z = \infty$, $V = 2\nu^{-1}\gamma^{-1}(1-\delta)$) which corresponds to the center of symmetry in physical space. For $\delta = 1$ we obtain the solution of Ia. B. Zel'dovich. In this case D = const, Q = const. The remaining parameters can be realized by assuming either that Q is proportional to D^2 or that $Q = Q_0 r^{2m}$, where $m = (\delta - 1)/\delta$.

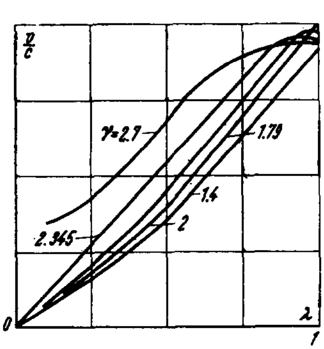

Fig. 3

We note that for $\delta_* < \delta \leq 1$ the acceleration of the gas particles at the detonation wave front is infinitely large, while for $\delta = \delta_*$ it is possible to have a divergent detonation wave under the Chapman-Jouguet conditions, in which case the acceleration at the detonation wave front is finite.

In the case of a convergent detonation wave for $\delta = \delta_*$ initial point (2.6) is singular (a node for $\gamma > 1$) and is associated with a positive and a negative value of dZ/dV. Considering $V = \delta_*$ as a limiting value for the smaller δ for which $dZ/dV > 0$, we choose the positive value of dZ/dV. Only on integral curve emerges from the singular point in the positive direction, so that the velocity of the gas particles behind the detonation wave decreases monotonously to zero along this curve. For $\delta = \delta_*$ the acceleration at the detonation wave front is finite. In paper [2] it is wrongly assumed that this acceleration is infinitely large. At the initial point the expression for the derivative is

$$\frac{d}{d\xi}\frac{v}{r_2} = (1 - \nu)\frac{2\gamma + 1 + k}{2 + k} \tag{2.8}$$

$$k = {}^1/_8\,(4\gamma - 5 \pm \sqrt{48\gamma^2 + 104\gamma + 105}) \qquad \text{for} \quad \nu = 2$$

$$k = {}^1/_5\,(\gamma - 2 \pm \sqrt{21\gamma^2 + 26\gamma + 54}) \qquad \text{for} \quad \nu = 3$$

The upper sign in (2.8) refers to a convergent detonation wave; the lower sign refers to a divergent wave.

For $\delta > \delta_*$ the acceleration at the detonation wave front is infinitely large. Initial point (2.6) in this case lies above the upper singular point on the parabola $Z = (V - \delta)^2$.

It should be noted that for Q proportional to D^2 the self-similarity index is not uniquely determined. But if we require that the acceleration of the gas particles at the detonation wave front be finite, then δ is unique. For $Q = Q_0 r^{2m}$ the index δ is unique.

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